1

Latent Variables in Multilevel Structural Equation Models

The examples of MSEM in the main text all use single indicators. Extending MSEM to latent variables is straightforward but there are some technical issues you should be aware of. Figure 1 shows a single mediator, single outcome RET model with three interchangeable indicators for the mediator and the outcome. I make my main points using this model.

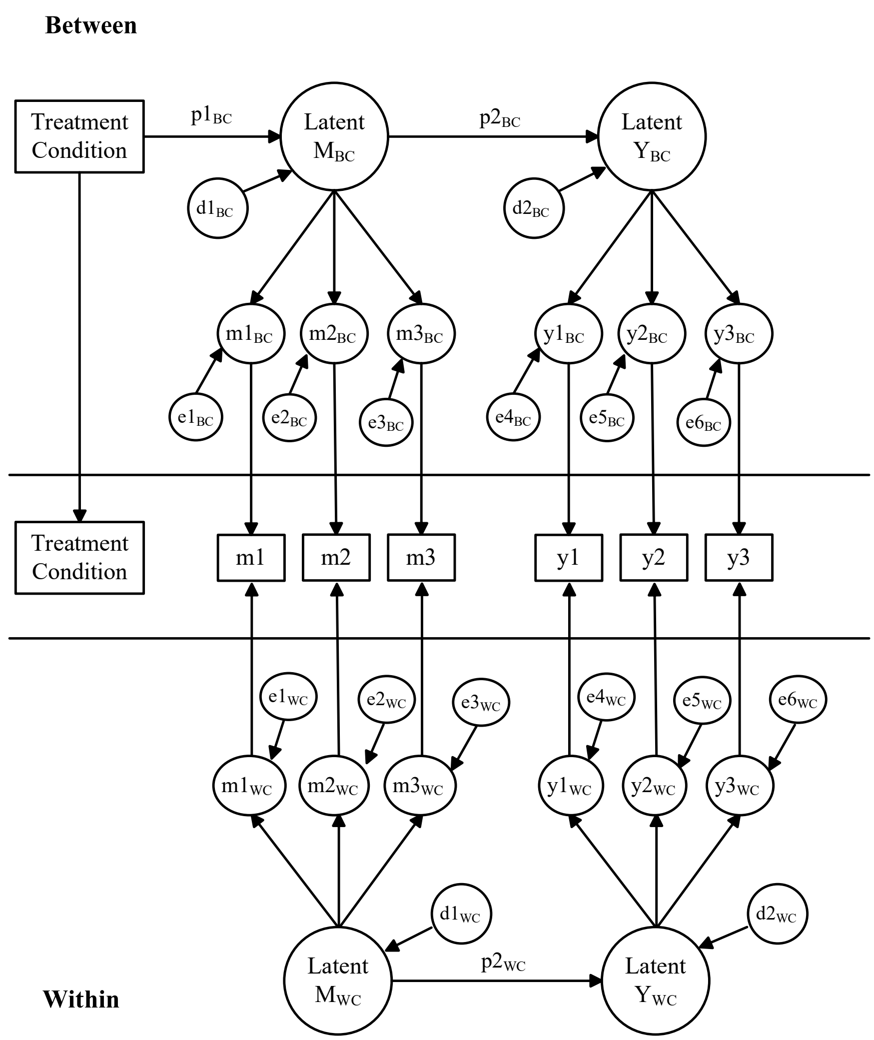

FIGURE 1. MSEM influence diagram with measurement multiple indicators

In general, latent variables can be used at the between-cluster level, at the within-cluster level, or both (Stapleton et al., 2016). The current example includes them at both levels. Suppose in a school based prevention program where schools are clusters, the latent Y variable in a clustered RET is depression with the indicators being 3 different self-ratings of depression on the part of students (y1, y2 and y3) with each measure ranging on a -5 to +5 metric. The indicators are thought to reflect the latent depression construct in the same way at both the between-cluster and within-cluster levels. Suppose one wants to compare the magnitude of $p2_{BC}$ with $p2_{WC}$ in Figure 1 to evaluate a context effect for the effect of the mediator on the outcome. Consistent with the concept of measurement invariance, to do so the unstandardized factor loadings of a given indicator should be equal (or functionally equivalent) at the across and within-cluster levels. Specifically, the loading for y1 at the between-cluster level should equal the loading for y1 at the within-cluster level; the loading for y2 at the between-cluster level should equal the loading for y2 at the within-cluster level; and the loading for y3 at the between-cluster level should equal the loading for y3 at the within-cluster level and such correspondence also should occur for the mediator (Asparouhov & Muthén, 2012; Mehta & Neale, 2005; Rabe-Hesketh et al., 2004). One can evaluate the viability of this assumption by testing the difference in values of the respective factor loadings at the two levels. Table 1 shows the Mplus syntax for such a test.

Table 1: Syntax for Test of Loading Invariance in MSEM

```
TITLE: MSEM invariance analysis ;
DATA: FILE IS invariance.dat ;
VARIABLE:
NAMES ARE
  y1 y2 y3 m1 m2 m3 treat school ;
USEVARIABLES ARE
  y1 y2 y3 m1 m2 m3 treat ;
CLUSTER is school ;
BETWEEN IS treat ;  ! specify global/integral level 2 variables
ANALYSIS:
TYPE = TWOLEVEL ;
ESTIMATOR = BAYES ;
BITERATIONS=100000 (50000); BCONVERGENCE =.01;
MODEL :
%WITHIN%
  y1 ; y2 ; y3 ;  ! estimate disturbance variances of outcome
  m1 ; m2 ; m3 ;  ! estimate variances of mediator
  lyw by y1 y2 y3 (pw1-pw3) ; ! define measurement model for y
  lmw by m1 m2 m3 (pw4-pw6) ; ! define measurement model for m
  lmw ; ! estimate variance of latent mediator
  lyw ; ! estimate disturbance variance of latent outcome
  lyw ON lmw (pw7) ; !regress latent outcome on latent mediator
```

```
%BETWEEN%
  y1 ; y2 ; y3 ; ! estimate residual variances of outcome
  m1 ; m2 ; m3 ; ! estimate residual variances of mediator
  lyb by y1 y2 y3 (pb1-pb3); ! define measurement model for y
  lmb by m1 m2 m3 (pb4-pb6); ! define measurement model for m
  lyb ; ! estimate disturbance variance of latent mediator
  lmb ; ! estimate disturbance variance of latent outcome
  lyb ON lmb (pb7) ; !regress latent outcome on latent mediator
  lmb on treat ; !regress latent mediator onto treatment condition
MODEL CONSTRAINT: ! conduct contrasts
NEW (ly2 ly3 lm2 lm3 context) ; ! give labels to contrasts
 ly2 = pw2-pb2 ; ! compare y indicator 2 loadings
 ly3 = pw3-pb3 ; ! compare y indicator 3 loadings
 lm2 = pw5-pb5 ; ! compare m indicator 2 loadings
 lm3 = pw6-pb6 ; ! compare m indicator 2 loadings
 context = pb7-pw7 ; ! test context effect
OUTPUT: STDYX CINTERVAL(HPD) RESIDUAL TECH4 TECH8 ;
```

Note that the syntax defines latent variables at both the within-cluster level and the between-cluster level. None of the syntax should be new to you. The test of loading invariance and the test of the context effect occurs under the `MODEL CONSTRAINT` command. I use the letters *b* and *w* in my labels to differentiate between-cluster and within-cluster parameters.

When I analyzed the data, I found that the largest PSR on the final iteration was 1.003, suggesting the model converged. Here is the relevant output for model fit:

```
MODEL FIT INFORMATION

Bayesian Posterior Predictive Checking using Chi-Square

          95% Confidence Interval for the Difference Between
          the Observed and the Replicated Chi-Square Values

                              -29.217            27.663

          Posterior Predictive P-Value              0.512
```

The confidence interval for the chi square statistic is reasonably symmetrical and contains the value of 0. The posterior predictive p-value is near 0.50. These results suggest reasonable model fit. I do not show them here but the model estimated correlations were all reasonably close to the model observed correlations at both the between-cluster and within-cluster levels, which also is consistent with a good fitting model.

Here are the unstandardized parameter estimates of interest:

```
MODEL RESULTS

                                Posterior  One-Tailed         95% C.I.
                    Estimate       S.D.      P-Value   Lower 2.5%  Upper 2.5%  Sig

Within Level

 LYW      BY
    Y1                 1.000       0.000      0.000       1.000       1.000
    Y2                 0.891       0.008      0.000       0.876       0.906      *
    Y3                 0.905       0.008      0.000       0.889       0.920      *

LMW      BY
    M1                 1.000       0.000      0.000       1.000       1.000
    M2                 0.853       0.010      0.000       0.834       0.873      *
    M3                 0.860       0.010      0.000       0.841       0.880      *

 LYW        ON
    LMW                0.503       0.013      0.000       0.478       0.528      *

Between Level

 LYB      BY
    Y1                 1.000       0.000      0.000       1.000       1.000
    Y2                 0.880       0.075      0.000       0.739       1.032      *
    Y3                 0.939       0.076      0.000       0.793       1.089      *

 LMB      BY
    M1                 1.000       0.000      0.000       1.000       1.000
    M2                 0.848       0.077      0.000       0.704       1.006      *
    M3                 0.833       0.081      0.000       0.681       0.997      *

 LYB        ON
    LMB                0.434       0.103      0.000       0.236       0.638      *

 LMB        ON
    TREAT              1.149       0.204      0.000       0.744       1.545      *

New/Additional Parameters
    LY2                0.011       0.075      0.441      -0.143       0.152
    LY3               -0.034       0.076      0.318      -0.190       0.109
    LM2                0.005       0.078      0.473      -0.156       0.149
    LM3                0.027       0.081      0.370      -0.140       0.179
    CONTEXT           -0.069       0.104      0.251      -0.276       0.131
```

The unstandardized factor loadings for both the outcome latent variable and the mediator latent variable are quite similar at the two levels of analysis, suggesting functional loading invariance. The first four rows of the section `New/Additional Parameters` are formal tests of across versus between loading differences for those loadings that were freely estimated (see Lines 34-37 in Table 1). As discussed in the measurement invariance document on my website for Chapter 3, this evaluation strategy assumes the first indicator of each latent variable is loading invariant; other strategies described in that document also could be pursued for sensitivity purposes.

The test for the presence of a context effect for the estimated effect of the latent mediator on the latent outcome also appears in the section `New/Additional Parameters` in the row labeled `CONTEXT`. The path coefficient reflecting the effect at the between-cluster level (coefficient = 0.44, 95% CI = 0.24 to 0.64) is not significantly different than the effect at the within-cluster level (coefficient = 0.50, 95% CI = 0.47 to 0.53). The difference in the coefficients is -0.07 whose credible interval (-0.28 to 0.13) contains the value of zero.

Analyses of latent variables in an MSEM framework sometimes are referred to as being **doubly robust** because they adjust for both measurement error as well as sampling error in the cluster sample means. It is not essential that measurement invariance for latent variables be demonstrated across levels at the within-cluster and between-cluster levels. If it does not exist, this simply means the respective latent variables may reflect different constructs and that it may not be meaningful to compare parallel path coefficients between the two levels for the target latent variable.

REFERENCES

Asparouhov, T., & Muthén, B. (2012). Multiple group multilevel analysis (mplus web notes no. 16). Retrieved from http://statmodel.com/exam ples/webnotes/webnote16.pdf

Mehta, P. D., & Neale, M. C. (2005). People are variables too: Multilevel structural equations modeling. *Psychological Methods*, 10, 259–284.

Rabe-Hesketh, S., Skrondal, A., & Pickles, A. (2004). Generalized multilevel structural equation modelling. *Psychometrika*, 69, 167–190.