Supplemental Instructions for Sample Size Determination for Contrasts

This document provides orienting information for you to use when conducting power analysis and margin of error analysis for the suite of programs focused on contrast analysis. These include the following programs: ‘Power: Contrasts’, ‘Power: One Sample’, ‘MOE: Contrasts’, and ‘MOE: One Sample’.

Background Information on Between-Subjects Contrasts

Many power analysis programs on the internet that focus on means provide power perspectives on omnibus tests. However, when it is all said and done, we almost always move to the analysis of single degree of freedom contrasts that underlie those omnibus tests. We need to ensure that these specific contrasts are sufficiently powered and that the margins of error for them are acceptable. I discuss such contrasts in Chapter 18 of my book. In this section, I describe the framework I use to conceptualize one way and factorial between-subject designs vis-a-vis contrast analysis.

I make use of a general formula to calculate the estimate of a contrast parameter value in a population based on means. I illustrate the formula here using a 2X2 factorial design. In practice, it can be applied to any size design, from one factor designs to five or more factor designs. In this example, suppose I examine the effects of grade (8th grade versus 9th grade) and sex (male versus female) on how satisfied adolescents are with their relationship with their parents (measured on a scale ranging from 0 to 100, with higher scores indicating more satisfaction). Let

M_{8M} = the mean for 8th grade males

M_{8F} = the mean for 8th grade females

M_{9M} = the mean for 9th grade males

M_{9F} = the mean for 9th grade females

I begin by forming an equation that weights (i.e., multiplies) each of the means by a “contrast coefficient,” as follows:

$$PE_1 = c_1 M_{8M} + c_2 M_{8F} + c_3 M_{9M} + c_4 M_{9F} \quad [1]$$

where PE_1 stands for “parameter estimate” and is the estimated contrast parameter value of interest. I assign values to the contrast coefficients to isolate comparisons I am interested in. For example, suppose I want to estimate the population mean difference between 8th grade males and 8th grade females. This is $M_{8M} – M_{8F}$. I can express this difference using Equation 1 by assigning the values $c_1 = 1$, $c_2= -1$, $c_3 = 0$, and $c_4 = 0$. This yields

$PE_1 = (1) M_{8M} + (-1) M_{8F} + (0) M_{9M} + (0) M_{9F}$

$= (1) M_{8M} + (-1) M_{8F}$

$= M_{8M} – M_{8F}$

Contrast analysis involves estimating the above parameter (i.e., the group mean difference), calculating a confidence interval and margin of error for it, and testing it for "statistical significance" in a null hypothesis sense. If I want to compare ninth grade males with ninth grade females, I use the coefficients c1 = 0, c2= 0, c3 = 1, and c4 = -1. This yields

$PE_2 = (0)\ M_{8M} + (0)\ M_{8F} + (1)\ M_{9M} + (-1)\ M_{9F}$

$= (1)\ M_{9M} + (-1)\ M_{9F}$

$= M_{9M} – M_{9F}$

Suppose I want to estimate the mean difference between 8th graders and 9th graders, collapsing across biological sex. In this case, the mean for 8th graders is $(M_{8M} + M_{8F})/2$ and for 9th graders it is $(M_{9M} + M_{9F})/2$. The difference between these is $[(M_{8M} + M_{8F})/2] – [(M_{9M} + M_{9F})/2]$. I can isolate this parameter with contrast coefficients. Focus first on the mean for 8th graders, $(M_{8M} + M_{8F})/2$. This can be re-written as

$(M_{8M} + M_{8F})/2 = ½\ (M_{8M} + M_{8F})$

$= 0.5\ (M_{8M} + M_{8F})$

$= 0.5\ M_{8M} + 0.5\ M_{8F}$

Using similar logic, the mean for 9th graders $(M_{9M} + M_{9F})/2$ can be rewritten as

$(M_{9M} + M_{9F})/2 = ½\ (M_{9M} + M_{9F})$

$= 0.5\ (M_{9M} + M_{9F})$

$= 0.5\ M_{9M} + 0.5\ M_{9F}$

and I can isolate the difference by assigning the values $c_1 = .5$, $c_2= .5$, $c_3 = -.5$, and $c_4 = -.5$. This yields

$PE_3 = (.5)\ M_{8M} + (.5)\ M_{8F} + (-.5)\ M_{9M} + (-.5)\ M_{9F}$

If we are creative, we can determine a set of coefficients that will isolate most any focused contrast of interest. For example, suppose I want to know if the male-female difference for 8th graders is the same as the male-female difference for 9th graders. The male-female difference for 8th graders is

$(1)\ M8_{M} + (-1)\ M_{8F}$

and the male-female difference for 9th graders is

$(1)\ M_{9M} + (-1)\ M_{9F}$

and the difference between these two differences is found by

$PE_4 = (1)\ M_{8M} + (-1)\ M_{8F} + (-1)\ M_{9M} + (1)\ M_{9F}$

In factorial designs, the three contrasts I just specified are called (1) a simple effect contrast (PE1), (2) a main effect contrast (PE3) and (3) an interaction contrast (PE4). The important point for now is that I can use the contrast formula approach to isolate estimates of a wide range of population contrasts that may be of interest. For factorial designs, we lay the means out for every cell in the design and then apply contrasts coefficients to the different cell means to isolate the contrast we are interested in. This approach to contrast analysis is very flexible and useful.

Note that PE_1 and PE_2 correspond to simple effects in a factorial design, PE_3 is a main effect and PE_4 is an interaction/moderation effect. In this sense, I can express any between-subject factorial design as a one way design and use carefully chosen contrast coefficients to isolate the contrasts I am interested in. I can readily use this framework for ANCOVA by conceptualizing the means as adjusted means.

Most researchers use contrast coefficients that preserve the metric of the outcome variable but doing so is not necessary for purposes of significance tests. For example, if for three groups one tests the difference between the first group and the average of the second two groups, one could use either of the following set of coefficients:

1, -.5, -.5

2, -1, -1

Both will yield the same result for a significance test, but the parameter estimate for the latter will be twice the size of the parameter estimate for the former because the metric of the outcome has been doubled. I personally prefer to define coefficients so that I retain the original metric.

An issue also arises when there are unequal n in the groups in the population. Suppose I want to compare the mean of the first group with the average of the second two groups where the means and sample sizes are as follows:

Group 1: Mean (M1) = 100 n1 = 100

Group 2: Mean (M2) = 50 n2 = 50

Group 3: Mean (M3) = 30 n3 = 25

There are two ways we can combine groups 2 and 3. First, I can average their two means, yielding (50 + 30)/2 = 40. This method ignores the sample size differences and yields what is called an unweighted mean. Second, I can calculate a weighted mean that takes into account the sample sizes (which would be the same as merging the respondents in the two groups together and calculating the mean on this combined sample size). In this case, the weighted mean equals 43.333. The most common approach to contrast analysis in the social sciences uses the unweighted mean method and that is what my programs focus on.

The contrast coefficients in the above example for unweighted means analysis are 1, -.5, and -.5. If I want to work with weighted means, I have to define the coefficients differently. For the first group, the coefficient would still be 1, but for the second two groups, the coefficients would take into account the respective expected sample sizes. The sum of the sample sizes for the two groups is 50 + 25 = 75. The sample size for the first group divided by this value (50/75) yields a coefficient of = .66667 and for the second group, it is 25/75 = 0.3333. So instead of using 1, -.5, -5 per the unweighted case, I would use for the weighted case 1, -.6667, -.3333. The choice of which method to use depends on your conceptual questions.

A requirement of contrast analysis as used in my program is that the coefficients must sum to 0. Note that such is the case for all the above examples.

Background Information on Within-Subjects Contrasts

Similar principles apply to designs that are completely within-subjects, i.e., repeated measures. Consider a one way repeated measures analysis of variance with three levels. Suppose you want to compare the average of the first two means against the third mean. For each individual, calculate a new variable (which I will call a "transformed outcome") defined as (Y1 + Y2)/2 - Y3. This maps onto the contrast of interest, namely for each individual, I average the scores for the first two conditions and then subtract the score for the third condition from that average. Then I can conduct a one-sample t test of the mean of this transformed outcome against a hypothesized value of zero. The resulting t statistic and its associated p value represents a test of the contrast.

For a completely within-subject factorial design, a simple computational approach to isolating contrasts is to define a new "transformed" Y variable score that combines the repeated measures in such a way that the transformed score corresponds to the contrast of interest; then conduct a one sample t test against zero on that transformed Y. For example, in a two-factor design that is all repeated measures, I might have the following:

	a_1	a_2
b_1	Y_1	Y_3
b_2	Y_2	Y_4

To evaluate the main effect of A, for each respondent calculate the score $[(Y_1 + Y_2)/ 2] - [(Y_3 + Y_4)/ 2]$. Then conduct a one sample t test on this transformed Y against a value of zero. To evaluate the main effect of B, for each respondent calculate the score $[(Y_1 + Y_3)/ 2] - [(Y_2 + Y_4)/ 2]$. Then conduct a one sample t test on this transformed Y against a value of zero. To evaluate the interaction contrast, define a score for each individual as $(Y_1 – Y_2) – (Y_3 – Y_4)$ and then conduct a one sample t test on this transformed Y against a value of zero. You can report the results as if you had applied more complex formulas from statistical design texts because the results will be the same (unless those texts used pooled error terms, which generally is not recommended; the above strategy uses unpooled error terms).

Based on the above, you can use power analysis and margin of error analysis as applied to the one sample t test to help inform sample size selection. This is why I provide the power and MOE programs for the one sample t test.

Background Information on Between-Within Contrasts

When one has a factorial design that has at least one between-subject factor and one within-subject factor, a common practice is to use pooled error terms for the between-subject factor(s), but unpooled error terms for the within-subject factors. This is because it is generally thought that significance tests with pooled error terms for between-subject factors are robust to violations of homogeneity of variance, but they are not robust to violations of sphericity for within-subject factors. Consider a two factor design where the between-subject factor is gender and the repeated measure factor is grade (7th, 8th and 9th; i.e., the same people are interviewed in grades 7, 8 and 9). The outcome variable is adolescents' satisfaction with their relationships with their parents. In such a design, researchers are typically interested in a variety of single degree of freedom contrasts. One type of contrast focuses on the main effect of gender collapsing across grade (the repeated measure factor). This test can be pursued using the computational trick described above: Calculate the average score across the three repeated measures for each individual in the study, $(Y_1 + Y_2 + Y_3)/3$. Then analyze this variable as a function of the between subject factor(s) in the design using standard between-subjects analysis of variance methods. The omnibus F test will be identical to the F test you would get from a traditional mixed factor analysis for the main effect of gender collapsing across grade. Conduct single degree of freedom contrasts for the between-subject factors on this transformed variable exactly as you would for a traditional between-subjects design in my program. The results you get will be identical to what you would get following traditional mixed-factor procedures in design textbooks.

As another example, if your design is a 3X2X3 factorial with the first two factors being between-subjects in nature and you want to analyze the outcome variable for the first two factors collapsing across the repeated measure factor, simply calculate a transformed Y score for each individual defined as $(Y_1 + Y_2 + Y_3)/3$. Then conduct a 3X2 between-subjects analysis of variance on the averaged scores. Pursue single degree of freedom contrasts using the methods I described above for a standard between-subjects analysis of variance. You will obtain the same results as recommended in statistical design books for testing contrasts in mixed factorial designs that collapse across the repeated measure factor.

If you want to perform pairwise contrasts on the repeated measure factor, simply conduct standard dependent groups t tests on each pair of the Y scores. If the between-subject factor has unequal n, this translates into a test of weighted means for the repeated measure factor.

You can see from the above how the suite of programs I offer can be used to help make sample size decisions for a wide range of contrasts. Watch the video for examples.